Question1

The radius of a cone of height 9 units is changed from 2 units to 2.12 units. The exact change and approximate change in the volume of the cone are respectively

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Options:

A.

$(1.4437)\pi, (1.44)\pi$

B.

$(1.4832)\pi, (1.479)\pi$

C.

$(1.4842)\pi, (1.48)\pi$

D.

$(1.4832)\pi, (1.44)\pi$

Answer: D

Solution:

$$\text{Volume of cone } (V) = \frac{1}{3}\pi r^2 h$$

$$\begin{aligned} h &= 9 \text{ units} \\ r_1 &= 2 \\ r_2 &= 2.12 \end{aligned}$$

$$\therefore \quad V_1 = \frac{1}{3}\pi\left(2^2 \times 9 = 12\pi \text{ cubic units}\right.$$

$$V_2 = \frac{1}{3}\pi(2.12)^2 \times 9$$

$= 3\pi(4.4944)$
$= 13.4832\pi$ cubic unit

$\therefore$ Exact change, $\Delta V = V_2 - V_1$

$= 13.4832 - 12\pi$
$= 1.4832\pi$ cubic units

Now, $\frac{dV}{dr} = \frac{1}{3}\pi(2r)(h) = \frac{2}{3}\pi rh$

$$\therefore \quad dV = \frac{dV}{dr}\Delta r = \left(\frac{2}{3}\pi rh\right)(0.12)$$

$$= \frac{2}{3}\pi(2)(9)(0.12)$$

$$= 1.44\pi \text{ cubic units.}$$

Question2

The local maximum value l and local minimum value m of $f(x) = \frac{x^2+2x+2}{x+1}$ in $R - \{-1\}$ exist at α, β respectively, then $\frac{l+m}{\alpha+\beta} =$

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Options:

A.

0

B.

-4

C.

-2

D.

2

Answer: A

Solution:

Given, $f(x)=\frac{x^2+2x+2}{x+1}$

$$\Rightarrow f'(x)=\frac{(x+1)(2x+2)-\left(x^2+2x+2\right)(1)}{(x+1)^2}$$
$$\Rightarrow f'(x)=\frac{\left(2x^2+2x+2x+2\right)-\left(x^2+2x+2\right)}{(x+1)^2}$$
$$=\frac{\left(x^2+2x\right)}{(x+1)^2}$$

For critical points, put $f'(x)=0$

$$\begin{aligned}&\Rightarrow \quad x^2+2x=0 \Rightarrow x(x+2)=0\\&\Rightarrow \quad x=0,-2\\&\therefore \quad \alpha=-2, \beta=0\end{aligned}$$

Now,

$$f''(x)=\frac{(x+1)^2\left(2x+2-2(x+1)\left(x^2+2x\right)\right.}{(x+1)^4}$$
$$=\frac{2}{(x+1)^3}$$

At $x=0, f''(x)>0$

$\Rightarrow \beta=0$ is the point of minima at $x=-2$

$f''(x)<0$

$\Rightarrow \alpha=-2$ is the point of maxima

$\therefore \quad l=\frac{(-2)^2+2(-2)+2}{(-2)+1}=\frac{4-4+2}{-1}=-2$

And $m=\frac{0+0+2}{0+1}=2$

$\therefore \frac{l+m}{\alpha+\beta}=\frac{-2+2}{-2+0}=0$

Question3

$P(5,2)$ is a point on the curve $y=f(x)$ and $\frac{7}{2}$ is the slope of the tangent to the curve at P. The area of the triangle (in sq. units)

formed by the tangent and the normal to the curve at P with X-axis is

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Options:

A.

35

B.

$\frac{35}{2}$

C.

$\frac{53}{7}$

D.

$\frac{53}{14}$

Answer: C

Solution:

Given, the slope of the tangent at $P(5,2)$ is $\frac{7}{2}$

$\therefore$ Equation of tangent at P is

$$\begin{aligned} & y-2=\frac{7}{2}(x-5) \\ \Rightarrow \quad & 2y-4=7x-35 \\ \Rightarrow \quad & 7x-2y=31 \end{aligned}$$

$\therefore$ The tangent intersect the X-axis,

$$\begin{aligned} & y=0 \\ \therefore \quad & 7x=31 \Rightarrow x=\frac{31}{7} \end{aligned}$$

$\therefore$ Tangent intersect the X-axis at $A\left(\frac{31}{7},0\right)$

Now, slope of normal at P is $-\frac{2}{7}$

Equation of normal at P is

$$y-2=-\frac{2}{7}(x-5)$$

$\Rightarrow \quad 7y-14=-2x+10 \Rightarrow 2x+7y=24$

$\therefore$ x-intercept of normal, $y=0$

$\Rightarrow \quad x=12$

$\Rightarrow$ The normal intersects the X-axis at $B(15,0)$

Base of the triangle $=\left|\left(12-\frac{31}{7}\right)\right|=\frac{53}{7}$

$$\begin{aligned}\therefore \text{ Area of triangle } &= \frac{1}{2}\times\frac{53}{7}\times 2\\ &= \frac{53}{7}\text{ sq. units}\end{aligned}$$

Question4

If a particle is moving in a straight line so that after t seconds its distance S (in cms) from a fixed point on the line is given by $S=f(t)=t^3-5t^2+8t$, then the acceleration of the particle at $t=5\text{sec}$ is (in cm/sec^2)

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Options:

A.

10

B.

30

C.

20

D.

40

Answer: C

Solution:

Given, $S = f(t) = t^3 - 5t^2 + 8t$

$$\Rightarrow \quad \text{Velocity } = \frac{dS}{dt} = t^2 - 10t + 8$$
$$\therefore \text{ Acceleration } = \frac{d^2S}{dt^2} = 6t - 10$$
at $\quad t = 5$

Acceleration of the particle

$$= 6 \times 5 - 10 = 30 - 10$$
$$= 20 \text{ cm/sec}^2$$

Question5

If $f : [a, b] \rightarrow [c, d]$ is a continuous and strictly increasing function, then $\frac{d-c}{b-a}$ is

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Options:

A.

value of the function at a point $t \in (a, b)$

B.

value of the function at $t \in (a, b)$ such that $f'(t) = 0$

C.

Slope of the tangent drawn to the curve $y = f(t)$ at a point $t \in (c, d)$

D.

Slope of the tangent drawn to the curve $y = f(t)$ at a point $t \in (a, b)$

Answer: D

Solution:

Given, $f : [a, b] \rightarrow [c, d]$ is a continuous and strictly increasing function.

$\therefore \quad f(a) = c, f(b) = d$

Now, $\frac{d-c}{b-a} = \frac{f(b)-f(a)}{b-a}$

$\Rightarrow$ Slope of tangent at a point $t \in (a, b)$

Question6

The acute angle between the curves $y = 3x^2 - 2x - 1$ and $y = x^3 - 1$ at their point of intersection which lies in the first quadrant is

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Options:

A.

$\tan^{-1}\left(\frac{2}{121}\right)$

B.

$\tan^{-1}(2)$

C.

$\tan^{-1}\left(\frac{1}{13}\right)$

D.

$\frac{\pi}{2}$

Answer: A

Solution:

Set the equations equal to each other

$$\begin{aligned} & 3x^2 - 2x - 1 = x^3 - 1 \\ \Rightarrow \quad & x^3 - 3x^2 + 2x = 0 \\ \Rightarrow \quad & x(x-1)(x-2) = 0 \\ \Rightarrow \quad & x = 0, x = 1 \text{ and } x = 2 \end{aligned}$$

For $x = 0, y = 0^3 - 1 = -1$, Point : $(0, -1)$

For $x = 1, y = 1^3 - 1 = 0$, Point : $(1, 0)$

For $x = 2, y = 2^3 - 1 = 7$, Point : $(2, 7)$

So, the point of intersection in the first quadrant is $(2, 7)$.

For, $y_1 = 3x^2 - 2x - 1$

So, $m_1 = \frac{dy_1}{dx} = 6x - 2$

At $(2, 7), m_1 = 6(2) - 2 = 12 - 2 = 10$

For $y_2 = x^3 - 1, m_2 = \frac{dy_2}{dx} = 3x^2$

At $(2, 7), m_2 = 3\left(2^2\right) = 12$

$\therefore \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \quad \left|\frac{10-12}{1+(10)(12)}\right|$

$\Rightarrow \quad \left|\frac{-2}{1+120}\right| \Rightarrow \left|\frac{-2}{121}\right|$

$\Rightarrow \tan\theta = \frac{2}{121} \Rightarrow \theta = \tan^{-1}\left(\frac{2}{121}\right)$

Thus, the acute angle between the curves at their point of intersection in the first quadrant is $\tan^{-1}\left(\frac{2}{121}\right)$.

Question7

If the rate of change of the slope of the tangent drawn to the curve $y = x^3 - 2x^2 + 3x - 2$ at the point $(2, 4)$ is k times the rate of change of its abscissa, then $k =$

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Options:

A.

2

B.

4

C.

6

D.

8

Answer: D

Solution:

Given, curve $y = x^3 - 2x^2 + 3x - 2$

So, slope of tangent, $m = \frac{dy}{dx}$

$= 3x^2 - 4x + 3$

Now, the rate of change of the slope of tangent $=$ second derivative $\frac{d^2y}{dx^2}$

$\Rightarrow \quad \frac{d}{dx}\left(3x^2 - 4x + 3\right) = 6x - 4$

At point $(2, 4)$,

$\frac{d^2y}{dx^2} = 6 \times 2 - 4 = 8$

Now, the abscissa is x. So, the rate of change of the abscissa is $\frac{dx}{dt}$

Now, given $\frac{d}{dt}\left(\frac{dy}{dx}\right) = k\frac{dx}{dt}$

$\Rightarrow \quad \frac{d^2y}{dx^2} \cdot \frac{dx}{dt} = k\frac{dx}{dt} \Rightarrow \frac{d^2y}{dx^2} = k$

$\Rightarrow \quad k = 8 \quad$ [using Eq. (i)]

Question8

If $f(x) = x + \log\left(\frac{x-1}{x+1}\right)$ is a well-defined real valued function, then f is

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Options:

A.

monotonically decreasing function

B.

monotonically increasing function

C.

increasing in $(1, \infty)$ and decreasing in $(-\infty, -1)$

D.

decreasing in $(1, \infty)$ and increasing in $(-\infty, -1)$

Answer: B

Solution:

Given, $f(x) = x + \log\left(\frac{x-1}{x+1}\right)$

For, $\log\left(\frac{x-1}{x+1}\right)$ to be well-defined, $\frac{x-1}{x+1} > 0$.

This inequality holds when

$\Rightarrow$ Both $x - 1 > 0$ and $x + 1 > 0 \Rightarrow x > 1$

$\Rightarrow$ Both $x - 1 < 0$ and $x + 1 < 0 \Rightarrow x < -1$

So, the domain of $f(x)$ is $(-\infty, -1) \cup (1, \infty)$

Now, $f'(x) = \frac{d}{dx}\left[x + \log\left(\frac{x-1}{x+1}\right)\right]$

$\Rightarrow 1 + \frac{(x+1)}{(x-1)} \cdot \frac{(x+1) \cdot 1 - 1 \cdot (x-1)}{(x+1)^2}$

$$\Rightarrow 1 + \frac{(x+1) - (x-1)}{(x-1)(x+1)}$$
$$\Rightarrow 1 + \frac{2}{(x-1)(x+1)}$$

Since, $x \in (-\infty, -1) \cup (1, \infty)$, we have $x^2 > 1$

$\Rightarrow x^2 - 1 > 0$

Thus, $\frac{2}{x^2-1} > 0$

So, $f'(x) = 1 + \frac{2}{x^2-1} > 1 + 0 = 1$

Since, $f'(x) > 0$ for all x in the domain, the function $f(x)$ is monotonically increasing.

Question9

A real valued function $f(x) = |x^2 - 3x + 2| + 2x - 3$ is defined on $[-2, 1]$. If m and M are absolute minimum and absolute maximum values of f respectively, then $M - 4m =$

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Options:

A.

0

B.

1

C.

15

D.

10

Answer: D

Solution:

Given, $f(x) = |x^2 - 3x + 2| + 2x - 3$ is defined on $[-2, 1]$

$x^2 - 3x + 2 = (x - 1)(x - 2)$

So, on the interval $[-2, 1]$

If $x \leq 1$, then $x - 1 \leq 0$

If $x \leq 2$ then $x - 2 \leq 0$

$\therefore$ On $[-2, 1]$, $(x - 1)(x - 2) \geq 0$

So, $|x^2 - 3x + 2| = x^2 - 3x + 2$ for $x \in [-2, 1]$

$$\therefore f(x) = (x^2 - 3x + 2) + 2x - 3$$
$$= x^2 - x - 1$$

Now, $f'(x) = 2x - 1$

For, critical points, set $f'(x) = 0$ we get

$$2x - 1 = 0$$
$$\Rightarrow \quad x = \frac{1}{2} \in [-2, 1]$$

Now, at $x = -2 f(-2) = (-2)^2 - (-2) - 1$

$\Rightarrow \quad 4 + 2 - 1 = 5$

At $\quad x = \frac{1}{2}, f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} - 1$

$\Rightarrow \quad \frac{1}{4} - \frac{1}{2} - 1 \Rightarrow \frac{1-2-4}{4} = -\frac{5}{4}$

At $\quad x = 1, f(1) = 1^2 - 1 - 1 = -1$

So, the absolute minimum value, $m = \frac{-5}{4}$ and the absolute maximum value, $M = 5$

$$\therefore M - 4m = 5 - 4 \times \left(-\frac{5}{4}\right)$$
$$\Rightarrow \quad 5 + 5 = 10$$

Question10

For a given function $y = f(x), \delta y$ denote the actual error in y corresponding to actual error δx in x and dy denotes the approximately value of δy. If $y = f(x) = 2x^2 - 3x + 4$ and $\delta x = 0.02$, then the value of $\delta y - dy$ when $x = 5$ is

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Options:

A. 0.0008

B. 0.008

C. 0.0004

D. 0.004

Answer: A

Solution:

Given, $y = f(x) = 2x^2 - 3x + 4$
$\delta_x = 0.02 \quad x = 5$

$\frac{dy}{dx} = 4x - 3$

$dy = (4x - 3)dx$

$dy = (4 \times 5 - 3)(0.02)$

$= 17 \times 0.02 = 0.34$

$\delta_y = f(x + \delta x) - yf(x)$

$\delta_y = f(5.02) - f(5)$

$= 2(5.02)^2 - 3(5.02) + 4 - \left(2(5)^2 - 3(5) + 4\right]$

$= 2\left[5.02)^2 - 5^2\right] - 3(5.02 - 5)$

$= 2(5.05 + 5)(5.02 - 5) - 3(0.02)$

$= 2(10.02)(0.02) - 0.06$

$= 4.008 - 0.06 = 0.3408$

$\delta y - dy = 0.3408 - 0.34 = 0.0008$

Question11

The length of the normal drawn at $t = \frac{\pi}{4}$ on the curve $x = 2(\cos 2t + t\sin 2t), y = 4(\sin 2t + t\cos 2t)$ is

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Options:

A. $\frac{4}{\pi}\sqrt{1+\pi^2}$

B. $4\sqrt{1+\pi^2}$

C. 4π

D. $\frac{4}{\pi}$

Answer: B

Solution:

$x=2(\cos 2t+t\sin 2t)$

$\frac{dx}{dt}=2[-\sin 2t\cdot 2+t\cos 2t\cdot 2+\sin 2t]$

$=2[2t\cos 2t-\sin 2t]$

$y=4(\sin 2t+t\cos 2t)$

$\frac{dy}{dt}=4[\cos 2t\cdot 2+t(-\sin 2t)2+\cos 2t]$

$=4\left[3\cos 2t-2t\sin^2 t\right]$

$\frac{dy}{dx}=\frac{4\left[3\cos 2t-2t\sin^2 t\right]}{2[2t\cos 2t-\sin 2t]}$

$\left(\frac{dy}{dx}\right)$ at $t=\frac{\pi}{4}=\frac{2\left[0-2\cdot\frac{\pi}{4}\right]}{\left[2\frac{\pi}{4}\times 0-1\right]}=\frac{-\pi}{-1}=\pi$

y at $t=\frac{\pi}{4}=4[1]=4$

$$\text{Length of normal }=|y_1|\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$=4\sqrt{1+\pi^2}$$

Question12

If Water is poured into a cylindrical tank of radius 3.5 ft at the rate of $1\text{cuft}/\text{min}$, then the rate at which the level of the water in the tank increases (in ft/min) is

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Options:

A. $\frac{1}{154}$

B. $\frac{8}{77}$

C. $\frac{2}{77}$

D. $\frac{1}{11}$

Answer: C

Solution:

To find the rate at which the water level rises in a cylindrical tank, given that water is poured at a constant rate, we start by using the formula for the volume of a cylinder:

$V = \pi r^2 h$

Here, V is the volume, r is the radius of the base, and h is the height of the water in the tank.

Given:

The radius $r = 3.5$ ft.

The rate of volume change (water being poured) $\frac{dV}{dt} = 1$ cu ft/min.

We want to find $\frac{dh}{dt}$, the rate at which the water level h is rising.

Differentiate both sides with respect to time t:

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

Substitute the values:

$1 = \pi(3.5)^2 \frac{dh}{dt}$

Solve for $\frac{dh}{dt}$:

$\frac{dh}{dt} = \frac{1}{\pi \times 3.5 \times 3.5}$

Simplifying further, we perform the multiplication:

$\pi =$ approximately $22/7$

$3.5 = \frac{7}{2}$

So we have:

$\frac{dh}{dt} = \frac{1}{(\frac{22}{7}) \times (\frac{7}{2}) \times (\frac{7}{2})}$

Simplify the expression:

$\frac{dh}{dt} = \frac{7}{22} \times \frac{10}{35} \times \frac{10}{35} = \frac{2}{77}$ ft/min

Therefore, the rate at which the water level in the tank rises is $\frac{2}{77}$ ft/min.

Question13

$y = 2x^3 - 8x^2 + 10x - 4$ is a function defined on [1,2]. If the tangent drawn at a point (a, b) on the graph of this function is parallel to X-

axis $a \in (1, 2)$, then $a =$

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Options:

A. 0

B. 5

C. 1

D. $\frac{5}{3}$

Answer: D

Solution:

$y = 2x^3 - 8x^2 + 10x - 4$

$\frac{dy}{dx} = 6x^2 - 16x + 10$

$\left(\frac{dy}{dx}\right)_{(a,b)} = 6a^2 - 16a + 10 = 0$

$3a^2 - 8a + 5 = 0$

$(3a - 5)(a - 1) = 0$

$a = \frac{5}{3}$ or 1

$1 < \frac{5}{3} < 2$

$a = \frac{5}{3}$

Question14

If m and M are respectively the absolute minimum and absolute maximum values of a function $f(x) = 2x^3 + 9x^2 + 12x + 1$ defined on $[-3, 0]$, then $m + M =$

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Options:

A. -7

B. 0

C. 1

D. 5

Answer: A

Solution:

To find the sum of the absolute minimum and maximum values of the function $f(x) = 2x^3 + 9x^2 + 12x + 1$ on the interval $[-3, 0]$, follow these steps:

Determine Critical Points:

Compute the derivative of $f(x)$:

$f'(x) = 6x^2 + 18x + 12 = 6(x^2 + 3x + 2) = 6(x + 2)(x + 1)$

Setting $f'(x) = 0$ yields critical points at $x = -2$ and $x = -1$.

Evaluate the Function at Critical Points and Endpoints:

Compute $f(x)$ at the critical points and endpoints of the interval:

$f(-3) = 2(-3)^3 + 9(-3)^2 + 12(-3) + 1 = -54 + 81 - 36 + 1 = -8$

$f(-2) = 2(-2)^3 + 9(-2)^2 + 12(-2) + 1 = -16 + 36 - 24 + 1 = -3$

$f(-1) = 2(-1)^3 + 9(-1)^2 + 12(-1) + 1 = -2 + 9 - 12 + 1 = -4$

$f(0) = 2(0)^3 + 9(0)^2 + 12(0) + 1 = 1$

Identify the Absolute Minimum and Maximum:

Compare the function values: $f(-3) = -8$, $f(-2) = -3$, $f(-1) = -4$, and $f(0) = 1$.

The absolute minimum is $m = -8$.

The absolute maximum is $M = 1$.

Calculate $m + M$:

$m + M = -8 + 1 = -7$

Therefore, $m + M = -7$.

Question15

The maximum interval in which the slopes of the tangents drawn to the curve $y = x^4 + 5x^3 + 9x^2 + 6x + 2$ increase is

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Options:

A. $\left[\frac{-3}{2}, -1\right]$

B. $\left[1, \frac{3}{2}\right]$

C. $R - \left[1, \frac{3}{2}\right]$

D. $R - \left[\frac{-3}{2}, -1\right]$

Answer: D

Solution:

To find the maximum interval where the slopes of the tangents to the curve $y = x^4 + 5x^3 + 9x^2 + 6x + 2$ are increasing, we need to consider the behavior of the derivative.

The slope of the tangent line is given by the first derivative, $\frac{dy}{dx}$. We need to determine when this slope function itself is increasing, which requires the second derivative to be positive.

Let's calculate the derivatives:

First derivative (slope of the tangent):

$\frac{dy}{dx} = 4x^3 + 15x^2 + 18x + 6$

Second derivative:

$\frac{d^2y}{dx^2} = 12x^2 + 30x + 18$

To find when this second derivative is greater than zero (indicating the slope function is increasing), solve the inequality:

$12x^2 + 30x + 18 > 0$

Factor the quadratic:

$2x^2 + 5x + 3 > 0$

$(x + \frac{3}{2})(x + 1) > 0$

This inequality holds true for:

$x \in \mathbb{R} - \left[\frac{-3}{2}, -1\right]$

Thus, the slopes of the tangents increase on the intervals outside of $\left[\frac{-3}{2}, -1\right]$.

Question16

If $A = \{P(\alpha, \beta) /$ the tangent drawn at P to the curve $y^3 - 3xy + 2 = 0$ is horizontal line $\}$ and $B = \{Q(a, b) /$ the tangent drawn at Q to the curve $y^3 - 3xy + 2 = 0$ is a vertical line $\}$, then $n(A) + n(B) =$

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Options:

A. 12

B. 1

C. 0

D. 4

Answer: B

Solution:

To understand the solution, let's analyze the conditions for horizontal and vertical tangents to the curve $y^3 - 3xy + 2 = 0$.

Analyzing Horizontal Tangents:

For a tangent at a point $P(\alpha, \beta)$ to be horizontal, the derivative y' must be zero. We find the derivative as follows:

$$3y^2 \frac{dy}{dx} - 3y - 3x \frac{dy}{dx} = 0$$

Solving for y':

$$y' = \frac{y}{y^2 - x}$$

For the tangent to be horizontal at (α, β), we set:

$$y'|_{\alpha,\beta} = \frac{\beta}{\beta^2 - \alpha} = 0$$

Which implies $\beta = 0$. Substituting $\beta = 0$ into the original curve equation:

$$0^3 - 3\alpha \cdot 0 + 2 = 0 \quad \Rightarrow \quad 2 = 0$$

This is a contradiction, so there are no points where the tangent is horizontal. Thus, $n(A) = 0$.

Analyzing Vertical Tangents:

For a tangent at a point $Q(a, b)$ to be vertical, the denominator of y' must be zero, leading to:

$b^2 - a = 0 \quad \Rightarrow \quad a = b^2$

Substituting $a = b^2$ back into the original curve equation:

$b^3 - 3b \cdot b^2 + 2 = 0$

$b^3 - 3b^3 + 2 = 0$

$-2b^3 + 2 = 0$

$b^3 = 1$

$b = 1$

Thus, $a = b^2 = 1^2 = 1$. Therefore, the point is $Q(1, 1)$, implying $n(B) = 1$.

Conclusion:

The number of points in sets A and B are:

$n(A) = 0$

$n(B) = 1$

Thus, the total $n(A) + n(B) = 1$.

Question17

$y = f(x)$ and $x = g(y)$ are two curves and $P(x, y)$ is a common point of the two curves. If at P on the curve $y = f(x)$, $\frac{dy}{dx} = Q(x)$ and at the same point P on the curve $x = g(y)$, $\frac{dx}{dy} = -Q(x)$, then

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Options:

A. the two curves have common tangent

B. the angle between two curves is 45°

C. tangent drawn at P to one curve is normal to the other curve at P

D. the two curves never intersect orthogonally

Answer: C

Solution:

Given the curves $y = f(x)$ and $x = g(y)$, and a common point $P(x, y)$ on both curves, we are tasked with evaluating their tangents and how they relate at this point.

For the curve $y = f(x)$, by differentiating both sides with respect to x, we have:

$\frac{dy}{dx} = f'(x) = Q(x)$

This is the slope of the tangent to the curve $y = f(x)$ at the point P.

Next, considering the curve $x = g(y)$, we differentiate with respect to y:

$\frac{dx}{dy} = g'(y) = -Q(x)$

This represents the slope of the tangent to the curve $x = g(y)$ at the same point P.

Now, see that

$\left(\frac{dy}{dx}\right) \times \left(\frac{dx}{dy}\right) = Q(x) \times (-Q(x)) = -1$

The product of the slopes being -1 indicates that the tangents are perpendicular. Therefore, the tangent line to one curve at point P serves as the normal line to the other curve at the same point. This confirms that the tangent of one curve is normal to the other at point P.

Question18

If the expression $7 + 6x - 3x^2$ attains its extreme value β at $x = \alpha$, then the sum of the squares of the roots of the equation $x^2 + \alpha x - \beta = 0$ is

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Options:

A. 21

B. -19

C. 19

D. -21

Answer: A

Solution:

Given, $7 + 6x - 3x^2$ or $-3x^2 + 6x + 1$

As coefficient of x^2 is less than 0 , then $f\left(\frac{-b}{2a}\right)$ is minima or maxima.

$\frac{-b}{2a} = \frac{-6}{2\times(-3)} = 1$

$f(1) = -3 + 6 + 7 = 10$

Here, $\alpha = 1, \beta = 10$

Let m and n be roots of $x^2 + x - 10 = 0$

$m + n = -1$ and $mn = -10$

$m^2 + n^2 = (m + n)^2 - 2mn = 21$

Question19

The equation of the normal drawn to the curve $y^3 = 4x^5$ at the point $(4, 16)$ is

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Options:

A. $20x + 3y = 128$

B. $20x - 3y = 32$

C. $3x - 20y + 308 = 0$

D. $3x + 20y = 332$

Answer: D

Solution:

We have, $y^3 = 4x^5$

$3y^2 \frac{dy}{dx} = 20x^4$

$\left[\frac{dy}{dx}\right]_{(4,16)} = \frac{20\times 4^4}{3\times(16)^2} = \frac{20}{3}$

Slope of normal $= -\frac{3}{20}$

Equation $3x + 20y = K$

$12 + 320 = K \Rightarrow 332 = K$

Frence, equation of normal is $3x + 20y = 332$

Question20

A point P is moving on the curve $x^3y^4 = 2^7$. The x-coordinate of P is decreasing at the rate of 8 units per second. When the point P is at $(2, 2)$, the y-coordinate of P

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Options:

A. increases at the rate of 6 units per second

B. decreases at the rate of 6 units per second

C. increases at the rate of 4 units per second

D. decreases at the rate of 4 units per second

Answer: A

Solution:

Given, $x^3y^4 = 2^7$

$3x^2y^4\frac{dx}{dt} + 4x^3y^3\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = \frac{-3x^2y^4}{4x^3y^3}\frac{dx}{dt} \quad \{\because \frac{dx}{dt} = -8\}$

$\Rightarrow \frac{dy}{dt} = -\frac{3}{4} \times \frac{2^2\times2^4}{2^3\times2^3} \times -8$

$\Rightarrow \frac{dy}{dt} = +6$

Question21

If the function $f(x) = x^3 + ax^2 + bx + 40$ satisfies the conditions of Rolle's theorem on the interval $[-5, 4]$ and $-5, 4$ are two roots of the equation $f(x) = 0$, then one of the values of c as stated in that theorem is

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Options:

A. 3

B. $\frac{1+\sqrt{67}}{3}$

C. $\frac{1+\sqrt{65}}{3}$

D. -2

Answer: B

Solution:

If $f(x)$ satisfies the condition of

Rolle's theorem

$$f(-5) = f(4) = 0$$

$$\Rightarrow \quad -125 + 25a - 5b = 64 + 16a + 4b$$

$$\Rightarrow \quad 9a - 9b = 189 \Rightarrow a - b = 21$$

$$\therefore \quad f(4) = 0$$

$$\Rightarrow \quad 64 + 16a + 4b + 40 = 0$$

$$\Rightarrow \quad 4a + b = -26$$

$$\Rightarrow \quad a - b = 21$$

On adding Eq. (i) and (ii), we get

$$5a = -5 \Rightarrow a = -1 \text{ and } b = -22$$

$$\therefore \quad f(x) = x^3 - x^2 - 22x + 40$$

$$f'(x) = 3x^2 - 2x - 22 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 264}}{6} \Rightarrow x = \frac{1 \pm \sqrt{67}}{3}$$

Question22

If x and y are two positive integers such that $x + y = 24$ and x^3y^5 is maximum, then $x^2 + y^2 =$

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Options:

A. 288

B. 296

C. 306

D. 320

Answer: C

Solution:

Given that x and y are 2 positive number.

Also, $x + y = 24 \Rightarrow x = 24 - y$

Let $s = x^3y^5 = (24 - y)^3y^5$

Now,

$\frac{ds}{dy} = 3(24 - y)^2 \times (-1) \times y^5 + (24 - y)^3 \times 5y^4$

$= (24 - y)^2 \cdot y^4[-3y + (24 - y)5]$

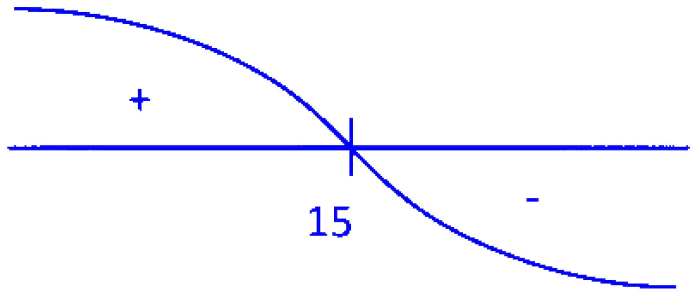
$= 8(24 - y)^2 \cdot y^4 \cdot (15 - y)$

$\frac{ds}{dy} = 0$ at $y = 0, 15, 24$

Now, applying first derivative test on 15 only, because x and y both are positive.

$\therefore$ x and y cannot be 0 so, y cannot be 0 and 24.

$\therefore$ Sign of $\frac{ds}{dy}$ around 15 is



$\therefore$ We have, S at maximum, when $y = 15$

So, $x = 24 - 15 = 9$

$\therefore x^2 + y^2 = 9^2 + 15^2 = 81 + 225 = 306$

Question23

If $4 + 3x - 7x^2$ attains its maximum value M at $x = \alpha$ and $5x^2 - 2x + 1$ attains its minimum value m at $x = \beta$, then $\frac{28(M-a)}{5(m+\beta)} =$

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Options:

A. 28

B. 23

C. 5

D. 1

Answer: B

Solution:

We have,

$4 + 3x - 7x^2$ attains its maximum value M at $x = \alpha$

$\therefore \quad \alpha = \frac{-(3)}{2(-7)} = \frac{3}{14}$

Now, $M = 4 + 3\left(\frac{3}{14}\right) - 7\left(\frac{3}{14}\right)^2 = \frac{121}{28}$

Also, we have $5x^2 - 2x + 1$ attains its minimum value M at $x = \beta$

$\therefore \quad \beta = \frac{-(-2)}{2(5)} = \frac{2}{10} = \frac{1}{5}$

Now, $m = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1 = \frac{4}{5}$

$$\therefore \quad \frac{28(M-\alpha)}{5(m+\beta)} = \frac{28\left(\frac{121}{28} - \frac{3}{14}\right)}{5\left(\frac{4}{5} + \frac{1}{5}\right)}$$

$$= \frac{115}{5} = 23$$

Question24

If $x = \cos 2t + \log(\tan t)$ and $y = 2t + \cot 2t$, then $\frac{dy}{dx} =$

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Options:

A. $\tan 2t$

B. $-\operatorname{cosec} 2t$

C. $-\cot 2t$

D. $\sec 2t$

Answer: B

Solution:

Given, $x = \cos 2t + \log(\tan t)$

and $\quad y = 2t + \cot 2t$

$$\therefore \quad \frac{dx}{dt} = (-\sin 2t)(2)$$

$$+\frac{1}{\tan t}\sec^2 t = -2\sin 2t + \frac{1}{\sin t\cos t}$$

$$\text{and } \frac{dy}{dx} = 2 - \left(\operatorname{cosec}^2 2t(2)\right.$$

$$= 2\left(1 - \operatorname{cosec}^2 2t\right)$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\left(1 - \operatorname{cosec}^2 2t\right)}{-2\sin 2t + \frac{1}{\sin t\cos t}}$$

$$= \frac{2\left(-\cot^2 2t\right)}{-2\sin 2t + \frac{2}{\sin 2t}}$$

$$= \frac{\frac{\cos^2 2t}{\sin^2 2t}}{\sin 2t - \frac{1}{\sin 2t}}$$

$$= \frac{\cos^2 2t}{(\sin^2 2t - 1)(\sin 2t)}$$

$$= \frac{\cos^2 2t}{-\cos^2 2t \sin 2t} = -\operatorname{cosec} 2t$$

Question25

The approximate value of $\sqrt[3]{730}$ obtained by the application of derivatives is

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Options:

A. 9.0041

B. 9.01

C. 9.006

D. 9.05

Answer: A

Solution:

Let $f(x) = \sqrt[3]{x}$

$\therefore \quad f'(x) = \frac{1}{3}(x)^{-2/3}$

We know that, $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$

$\therefore \qquad \sqrt[3]{730} = \sqrt[3]{729 + 1}$

$\approx \sqrt[3]{729} + \frac{1}{3}(729)^{-2/3}$

$= 9 + \frac{1}{3\times 81} \approx 9 + 0.0041$

$= 9.0041$

Hence, the approximate value of $\sqrt[3]{730}$ is 9.0041

Question26

If θ is the acute angle between the curves $y^2 = x$ and $x^2 + y^2 = 2$, then $\tan\theta =$

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Options:

A. 1

B. 3

C. 2

D. 4

Answer: B

Solution:

Given curves are $y^2 = x$ and

$x^2 + y^2 = 2$

The points of intersection of these curves are $(1,1)$ and $(1,-1)$

Now,

$y^2 = x$	$x^2 + y^2 = 2$
$2y\frac{dy}{dx} = 1$	$2x + 2y\frac{dy}{dx} = 0$
$\frac{dy}{dx} = \frac{1}{2y}$	$\frac{dy}{dx} = -\frac{x}{y}$
$\left(\frac{dy}{dx}\right)_{(1,1)} = \frac{1}{2(1)} = \frac{1}{2}$	$\left(\frac{dy}{dx}\right)_{(1,1)} = -\frac{1}{1} = -1$
$m_1 = \frac{1}{2}$	$m_2 = -1$

$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \because \theta$ is acute

$= \left|\frac{\frac{1}{2} - (-1)}{1 + \left(\frac{1}{2}\right)(-1)}\right| = \left|\frac{\frac{3}{2}}{\frac{1}{2}}\right| = 3$

Question27

The vertical angle of a right circular cone is 60°. If water is being poured in to the cone at the rate of $\frac{1}{\sqrt{3}}\ \mathrm{m}^3/\mathrm{min}$, then the rate (m/min) at which the radius of the water level is increasing when the height of the water level is 3 m is

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Options:

A. $\frac{1}{3\sqrt{3\pi}}$

B. $\frac{1}{9\sqrt{3\pi}}$

C. $\frac{1}{9\pi}$

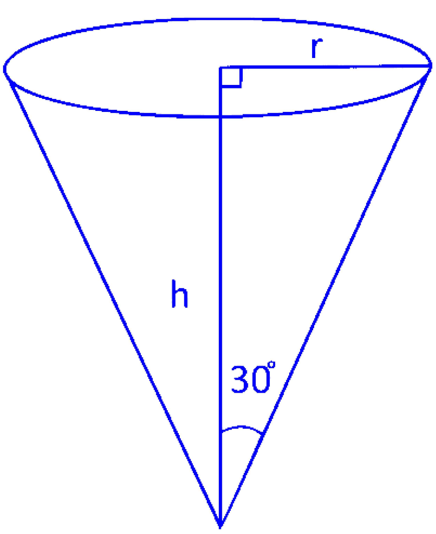
D. $\frac{1}{3\pi}$

Answer: C

Solution:

Semi-vertical angle $= \frac{60^{\circ}}{2} = 30^{\circ}$

$\tan 30^{\circ} = \frac{r}{h}$



$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{r}{h}$

$\Rightarrow \quad h=\sqrt{3} r$

When $h=3 m$, then $r=\frac{3}{\sqrt{3}}=\sqrt{3} m$

Volume, $V=\dfrac{1}{3} \pi r^2 h$

$\Rightarrow \quad 3 V=\pi r^2(\sqrt{3} r)$

$\Rightarrow \quad \sqrt{3} V=\pi r^3$

On differentiating Both sides w.r.t. t, we get

$$\sqrt{3} \frac{d V}{d t}=\pi\left(3 r^2\right) \frac{d r}{d t}$$

$$\Rightarrow \sqrt{3} \times \frac{1}{\sqrt{3}}=3 r^2 \pi \frac{d r}{d t}$$

$$\Rightarrow \left.\frac{d r}{d t}\right|_{h=3 m}=\frac{1}{3(\sqrt{3})^2 \pi}=\frac{1}{9 \pi} \mathrm{~m} / \mathrm{min}$$

Semi-vertical angle $=\frac{60^{\circ}}{2}=30^{\circ}$

$\tan 30^{\circ}=\frac{r}{h}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{r}{h}$

$\Rightarrow \quad h=\sqrt{3} r$

When $h=3 m$, then $r=\frac{3}{\sqrt{3}}=\sqrt{3} m$

Volume, $V=\dfrac{1}{3} \pi r^2 h$

$\Rightarrow \quad 3 V=\pi r^2(\sqrt{3} r)$

$\Rightarrow \quad \sqrt{3} V=\pi r^3$

On differentiating Both sides w.r.t. t, we get

$$\sqrt{3} \frac{d V}{d t}=\pi\left(3 r^2\right) \frac{d r}{d t}$$

$$\Rightarrow \sqrt{3} \times \frac{1}{\sqrt{3}}=3 r^2 \pi \frac{d r}{d t}$$

$$\Rightarrow \left.\frac{d r}{d t}\right|_{h=3 m}=\frac{1}{3(\sqrt{3})^2 \pi}=\frac{1}{9 \pi} \mathrm{~m} / \mathrm{min}$$

Question28

A right circular cone is inscribed in a sphere of radius 3 units. If the volume of the cone is maximum, then semi-vertical angle of the cone is

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Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\tan^{-1}(\sqrt{2})$

D. $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

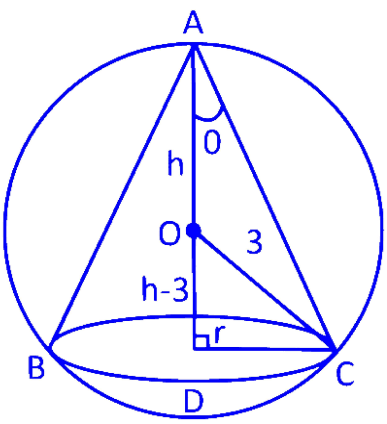
Answer: D

Solution:

Let r and h be radius and height of the cone.

Let the semi-vertical angle be θ.

Here, $\theta = \tan^{-1}\left(\frac{r}{h}\right)$



in right-angle triangle, ODC

$$3^2 = (h-3)^2 + r^2$$

$$\Rightarrow r^2 = 9 - \left(h^2 + 9 - 6h\right)$$

$$\Rightarrow r^2 = 6h - h^2$$

Volume of cone, $V=\frac{1}{3}\pi\left(6h-h^2\right)h$

$V=\frac{1}{3}\pi\left(6h^2-h^3\right)$

$\therefore \frac{dV}{dh}=\frac{1}{3}\pi\left(12h-3h^2\right)$

and $\frac{d^2V}{dh^2}=\frac{1}{3}\pi(12-6h)$

On setting $\frac{dV}{dh}=0$, we get $h=4$

For $h=4, \frac{d^2V}{dh^2}<0$

$\therefore$ Maximum volume will be occurs at $h=4$

Now, $r^2=6h-h^2=6(4)-4^2=8$

$\Rightarrow \quad r=2\sqrt{2}$

Hence, $\theta=\tan^{-1}\left(\frac{r}{h}\right)$

$=\tan^{-1}\left(\frac{2\sqrt{2}}{4}\right)=\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Question29

If $f(x)=kx^3-3x^2-12x+8$ is strictly decreasing for all $x\in R$, then

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Options:

A. $k<-\frac{1}{4}$

B. $k>-\frac{1}{4}$

C. $k>\frac{1}{4}$

D. $k<\frac{1}{4}$

Answer: A

Solution:

Given,

$f(x) = kx^3 - 3x^2 - 12x + 8$ is strictly

decreasing, $\forall x \in R$

$\therefore f'(x) = 0$ will not have any real solution.

Now, $f'(x) = 3kx^2 - 6x - 12$

$\because 3kx^2 - 6x - 12 = 0$ does not have any real solution.

$$\begin{aligned} \therefore \quad & D < 0 \\ \Rightarrow \quad & (-6)^2 - 4(3k)(-12) < 0 \\ \Rightarrow \quad & 36 + 36(4k) < 0 \Rightarrow 1 + 4k < 0 \\ \Rightarrow \quad & k < -\frac{1}{4} \end{aligned}$$

Question30

The radius of a sphere is 7 cm . If an error of 0.08 sq cm is made in measuring it, then the approximate error (in cubic cm) found in its volume is

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Options:

A. 0.28

B. 0.32

C. 0.96

D. 0.098

Answer: A

Solution:

We have,

radius of sphere $= 7$ cm

Error in area $= 0.08 \text{ cm}^2$ [given]

i.e. $\Delta A = 0.08 \text{ cm}^2$

We need to find

$$\Delta V = \frac{dV}{dA} \cdot \Delta A \quad \ldots (1)$$

$$\because A = 4\pi r^2 \Rightarrow \frac{dA}{dr} = 8\pi r$$

and $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2$

So, $\frac{dV}{dA} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2} \Rightarrow \left.\frac{dV}{dA}\right|_{r=7} = \frac{7}{2}$

Hence, $\Delta V = \frac{7}{2} \times 0.08 \quad \ldots$ [from Eq. (i)]

$= 0.28 \text{ cm}^3$

Approximate error in its volume $= 0.28 \text{ cm}^3$.

Question31

The curve $y = x^3 - 2x^2 + 3x - 4$ intersects the horizontal line $y = -2$ at the point $P(h, k)$. If the tangent drawn to this curve at P meets the X-axis at (x_1, y_1), then $x_1 =$

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Options:

A. 1

B. 2

C. 3

D. -3

Answer: B

Solution:

We have, curve $y = x^3 - 2x^2 + 3x - 4$ intersects the line

$y = -2$ at $P(h, k)$

$\Rightarrow \quad k = -2$ and $-2 = h^3 - 2h^2 + 3h - 4$

$\Rightarrow \quad h^3 - 2h^2 + 3h - 2 = 0$

$(h - 1)\left(h^2 - h + 2\right) = 0 \Rightarrow h = 1$

So, point $P \equiv (1, -2)$

$\left.\frac{dy}{dx}\right|_{\text{at } P} = 3x^2 - 4x + 3 = 2$

Now, equation of tangent at $P(1, -2)$ is

$y + 2 = 2(x - 1)$

$\Rightarrow \quad 2x - y - 4 = 0 \quad \ldots$ (i)

$\because$ Tangent at P meets X-axis at (x_1, y_1)

$\Rightarrow \quad y_1 = 0$

From Eq. (i), we get

$2x_1 - 0 - 4 = 0 \Rightarrow x_1 = \frac{4}{2} = 2$

Question32

If $f(x) = (2x - 1)(3x + 2)(4x - 3)$ is a real valued function defined on $\left[\frac{1}{2}, \frac{3}{4}\right]$, then the value(s) of c as defined in the statement of Rolle's theorem

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Options:

A. does not exist

B. $\frac{7 \pm \sqrt{247}}{36}$

C. $\frac{7 - \sqrt{247}}{36}$

D. $\frac{7 + \sqrt{247}}{36}$

Answer: D

Solution:

We have,

$f(x) = (2x - 1)(3x + 2)(4x - 3)$

$f'(x) = 2(3x + 2)(4x - 3)$

$+ 3(2x - 1)(4x - 3)$

$+ 4(2x - 1)(3x + 2)$

$= 2\left[12x^2 - x - 6\right] + 3\left[8x^2 - 10x + 3\right]$

$+4\left[6x^2 + x - 2\right]$

$= 24x^2 - 2x - 12 + 24x^2 - 30x + 9$

$+$

$+ 24x^2 + 4x - 8$

$f'(x) = 72x^2 - 28x - 11$

For Rolle's theorem, c is such that

$f'(c) = 0, \text{ where } c \in \left[\frac{1}{2}, \frac{3}{4}\right]$

$\Rightarrow \quad 72c^2 - 28c - 11 = 0$

$$c = \frac{28 \pm \sqrt{(28)^2 + 4 \times 11 \times 72}}{2 \times 72}$$

$$\because \quad c = \frac{7 \pm \sqrt{247}}{36}$$

$$\because \quad c \in \left[\frac{1}{2}, \frac{3}{4}\right] \Rightarrow c = \frac{7 + \sqrt{247}}{36}$$

Question33

If the interval in which the real valued function $f(x) = \log\left(\frac{1+x}{1-x}\right) - 2x - \frac{x^3}{1-x^2}$ is decreasing in (a, b), where $|b - a|$ is maximum, then $\frac{a}{b} =$

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Options:

A. -1

B. 1

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: A

Solution:

We have,

$$f(x)=\log\left(\frac{1+x}{1-x}\right)-2x-\frac{x^3}{1-x^2}$$

$$f'(x)=\left(\frac{1-x}{1+x}\right)\left[\frac{1-x-(1+x)(-1)}{(1-x)^2}\right]$$

$$-2-\left[\frac{\left(1-x^2\right)\cdot 3x^2-x^3(-2x)}{\left(1-x^2\right)^2}\right]$$

$$=\left(\frac{1-x}{1+x}\right)\left[\frac{1-x+1+x}{(1-x)^2}\right]$$

$$-2-\left[\frac{3x^2-3x^4+2x^4}{\left(1-x^2\right)^2}\right]$$

$$=\frac{2}{1-x^2}-2-\frac{3x^2-x^4}{\left(1-x^2\right)^2}$$

$$=\frac{2\left(1-x^2\right)-2\left(1-x^2\right)^2-3x^2+x^4}{\left(1-x^2\right)^2}$$

$$=\frac{2-2x^2-2-2x^4+4x^2-3x^2+x^4}{\left(1-x^2\right)^2}$$

$$=\frac{-x^4-x^2}{\left(1-x^2\right)^2}=\frac{-x^2\left(x^2+1\right)}{(1-x)^2(1+x)^2}$$

$\because f(x)$ is decreasing

$$\therefore f'(x)\leq 0\Rightarrow\frac{-x^2\left(x^2+1\right)}{(1-x)^2(1+x)^2}\leq 0$$

$$\Rightarrow\frac{x^2\left(x^2+1\right)}{(1-x)^2(1+x)^2}\geq 0$$

$x\in R-\{-1,1\}$

Question34

If the slope of the tangent drawn at any point (x,y) on the curve $y=f(x)$ is $\left(6x^2+10x-9\right)$ and $f(2)=0$, then $f(-2)=$

Options:

A. 0

B. 4

C. -6

D. -13

Answer: B

Solution:

We have slope of tangent

$= 6x^2 + 10x - 9$

$\Rightarrow \frac{dy}{dx} = 6x^2 + 10x - 9$

$dy = (6x^2 + 10x - 9) dx$

On integrating both sides, we get

$\int dy = \int (6x^2 + 10x - 9) dx$

$y = 2x^3 + 5x^2 - 9x + C$

$\because f(2) = 0$

$\Rightarrow 0 = 16 + 20 - 18 + C \Rightarrow C = -18$

So, $y = 2x^3 + 5x^2 - 9x - 18 = f(x)$

$\therefore f(-2) = 2(-2)^3 + 5(-2)^2 - 9(-2) - 18$

$= -16 + 20 + 18 - 18 = 4$

Question35

For all real values of x, the minimum value of $\frac{1-x+\lambda^2}{1+x+x^2}$ is

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Options:

A. 0

B. $\frac{1}{3}$

C. 1

D. 3

Answer: B

Solution:

Given the function $f(x)=\frac{1-x+x^2}{1+x+x^2}$, we want to find its minimum value for all real values of x.

First, differentiate $f(x)$ with respect to x:

$f'(x)=\frac{(2x-1)(x^2+x+1)-(2x+1)(x^2-x+1)}{(x^2+x+1)^2}$

Simplify the expression:

$$=\frac{(2x^3-x^2+2x^2-x+2x-1)-(2x^3-2x^2+2x^2+x-x+1)}{(x^2+x+1)^2}$$
$$=\frac{-x^2+2x^2+x-2x-1}{(x^2+x+1)^2}$$
$$=\frac{2x^2-2}{(x^2+x+1)^2}$$

To find the critical points, set $f'(x)=0$:

$2x^2-2=0x^2=1x=\pm1$

Evaluate $f(x)$ at $x=1$ and $x=-1$:

At $x=1$:

$f(1)=\frac{1-1+1^2}{1+1+1^2}=\frac{1}{3}$

At $x=-1$:

$f(-1)=\frac{1-(-1)+1^2}{1-1+1^2}=3$

Hence, the minimum value of $f(x)$ is $\frac{1}{3}$.

Question36

Electric current (I) is measured by galvanometer, the current being proportional to the tangent of the angle (θ) of deflection. If the deflection is read as 45° and an error of 1% is made in reading it, the percentage error in the current is

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Options:

A. π

B. $\pi/2$

C. $\pi/3$

D. $\pi/4$

Answer: D

Solution:

Approximate error in current

$dI = f'(I) \times \Delta I = \frac{\pi}{4} \times 0.01$

$\therefore$ Percentage error in current

$= \left(\frac{\pi}{4} \times 0.01\right) \times 100 = \frac{\pi}{4}$

Question37

If the equation of a tangent drawn to the curve $y = \cos(x + y), -1 \leq x \leq 1 + \pi$ is $x + 2y = k$, then $k =$

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Options:

A. 1

B. $\pi/4$

C. $\pi/2$

D. 2

Answer: C

Solution:

Given that, $y = \cos(x+y) \qquad \ldots(i)$

Differentiating on both side w.r.t. x, we get

$\frac{dy}{dx} = -\sin(x+y)\left(1+\frac{dy}{dx}\right)$

Equation of tangent is $x + 2y = k \qquad \ldots(ii)$

$\Rightarrow$ Slope is $-\frac{1}{2} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

Hence, $-\frac{1}{2} = -\sin(x+y)\left(1-\frac{1}{2}\right)$

$\Rightarrow \quad -\frac{1}{2} = -\frac{1}{2}\sin(x+y)$

$\Rightarrow \sin(x+y) = 1 = \sin\frac{\pi}{2}$

$\Rightarrow \quad x+y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - x$

From Eq. (i), we get

$$\frac{\pi}{2} - x = \cos\left(x + \frac{\pi}{2} - x\right)$$
$$\Rightarrow \frac{\pi}{2} - x = \cos\left(\frac{\pi}{2}\right) = 0$$
$$\Rightarrow \quad x = \frac{\pi}{2} \Rightarrow y = 0$$

Now, from Eq. (ii), we get

$k = \frac{\pi}{2} + 0 \cdot 2 = \frac{\pi}{2}$

Question38

$f: R \to R$ is a function defined by $f(x)=\frac{1}{e^x+2e^{-x}}$

Assertion (A) : $f(c)=\frac{1}{3}$ for some values of $c \in R$

Reason (R) : $0<f(x) \leq \frac{1}{2\sqrt{2}}$ for all $x \in R$

Then, which of the following options is correct?

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Options:

A. (A) and (R) are true, (R) is the correct explanation of (A)

B. (A) and (R) are true, (R) is not the correct explanation for (A)

C. (A) is true but (R) is false

D. (A) is false but (R) is true

Answer: A

Solution:

Given that,

$f(x)=\frac{1}{e^x+2e^{-x}}=\frac{e^x}{e^{2x}+2}$

Differentiating on both sides w.r.t. x, we get

$f'(x)=\frac{\left(e^{2x}+2\right)e^x-2e^{2x}e^x}{\left(e^{2x}+2\right)^2}$

Now, $f'(x)=0 \Rightarrow \frac{\left(e^{2x}+2\right)e^x-2e^{3x}}{\left(e^{2x}+2\right)^2}=0$

$$\begin{aligned} \Rightarrow \quad & 2e^x-e^{3x}=0 \\ \Rightarrow \quad & e^x\left(e^{2x}-2\right)=0 \end{aligned}$$

Since, $e^x \neq 0 e^{2x}=2 \Rightarrow e^x=\sqrt{2}$

For maximum

$f(x)=\frac{\sqrt{2}}{2+2}=\frac{\sqrt{2}}{4}=\frac{1}{2\sqrt{2}}$

Thus, $0

$\Rightarrow C \in R$ such that, $f(c)=\frac{1}{3}$

So, Assertion (A) is true.

and Reason (R) is true.

and Reason (R) is the correct explanation of Assertion (A).

Question39

If the expression $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x-\alpha)^2(x-\beta)$, then the values of λ are

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Options:

A. $27, -5$

B. $-27, -5$

C. 27,5

D. $-27, 5$

Answer: D

Solution:

Given the expression $f(x) = x^3 + 3x^2 - 9x + \lambda$, we need to find the values of λ such that $f(x)$ can be expressed as $(x-\alpha)^2(x-\beta)$.

Since $(x-\alpha)$ is a repeated factor, it must also be a factor of the derivative $f'(x)$. We compute the derivative:

$f'(x) = 3x^2 + 6x - 9$

Factor the derivative:

$f'(x) = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$

This shows that the potential values of α are the roots of $f'(x) = 0$, which are $\alpha = 1$ and $\alpha = -3$.

Case 1: $\alpha = 1$

Substitute $\alpha = 1$ into the original expression to find λ:

$f(1) = 1^3 + 3(1)^2 - 9(1) + \lambda = 0$

$1 + 3 - 9 + \lambda = 0$

$\lambda = 5$

Case 2: $\alpha = -3$

Substitute $\alpha = -3$ into the original expression to find λ:

$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + \lambda = 0$

$-27 + 27 + 27 + \lambda = 0$

$\lambda = -27$

Hence, the values of λ can be 5 or -27.

Question40

The equation of the normal at $t = \frac{\pi}{2}$ to the curve $x = 2\sin t, y = 2\cos t$ is

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Options:

A. $x = 2$

B. $y = 2x + 3$

C. $y = 0$

D. $y = 3$

Answer: B

Solution:

The given parametric equations for the curve are $x = 2\sin t$ and $y = 2\cos t$.

To find the equation of the normal line at $t = \frac{\pi}{2}$, we first need to determine the slope of the tangent line. Using the derivatives:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{2\cos t}$

This expression represents the slope of the tangent line to the curve.

Next, the slope of the normal line, which is perpendicular to the tangent line, is given by:

Slope of normal $= -\frac{1}{\frac{-2\sin t}{2\cos t}} = \frac{\cos t}{\sin t}$

Substituting $t = \frac{\pi}{2}$:

$m_s = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$

Thus, the slope of the normal line at $t = \frac{\pi}{2}$ is 0, which indicates a horizontal line.

The point on the curve at $t = \frac{\pi}{2}$ is:

$x = 2\sin\frac{\pi}{2} = 2, \quad y = 2\cos\frac{\pi}{2} = 0$

The equation of a horizontal line through this point, where the slope ($m = 0$), is:

$(y - 2\cos t) = 0(x - 2\sin t)$

Simplifying this at $t = \frac{\pi}{2}$:

$\left(y - 2 \times \cos\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad y = 0$

Thus, the equation of the normal line at $t = \frac{\pi}{2}$ is $y = 0$.

Question41

If the function $f(x) = \frac{x}{5} + \frac{5}{x}, (x \neq 0)$ attains its relative maximum value at $x = \alpha$, then $\sqrt{\alpha^2 + 2\alpha - 6} =$

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Options:

A. 10

B. 6

C. 5

D. 3

Answer: D

Solution:

To find the relative maximum of the function $f(x) = \frac{x}{5} + \frac{5}{x}$, where $x \neq 0$, follow these steps:

Derive the Function:

Compute the first derivative:

$f'(x) = \frac{1}{5} - \frac{5}{x^2}$

Set the derivative equal to zero for critical points:

$\frac{1}{5} - \frac{5}{x^2} = 0 \quad \Rightarrow \quad \frac{5}{x^2} = \frac{1}{5}$

Solve for x:

$x^2 = 25 \quad \Rightarrow \quad x = \pm 5$

Second Derivative Test:

Compute the second derivative:

$f''(x) = \frac{10}{x^3}$

Evaluate the second derivative at $x = 5$:

$f''(5) = \frac{10}{125} = \frac{2}{25} > 0$

Since $f''(5) > 0$, $x = 5$ is a point of relative minimum.

Evaluate the second derivative at $x = -5$:

$f''(-5) = \frac{-10}{125} = -\frac{2}{25} < 0$

Since $f''(-5) < 0$, $x = -5$ is a point of relative maximum.

Calculate the Expression:

Given that $\alpha = -5$, calculate:

$\sqrt{\alpha^2 + 2\alpha - 6} = \sqrt{(-5)^2 + 2(-5) - 6}$

Simplify:

$= \sqrt{25 - 10 - 6} = \sqrt{9} = 3$

Therefore, $\sqrt{\alpha^2 + 2\alpha - 6} = 3$.
